Local Grand Unification and String Theory

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Abstract. The low energy effective action of string theory depends strongly on the process of compactification and the localization of fields in extra dimensions. Explicit string constructions towards the minimal supersymmetric standard model (MSSM) reveal interesting results leading to the concept of local grand unification. Properties of the MSSM indicate that we might live at a special location close to an orbifold fixed point rather than a generic point in Calabi-Yau moduli space. We observe an enhancement of (discrete) symmetries that have various implications for the properties of the MSSM such as proton stability as well as solutions to the flavor problem, the μ -problem and the strong CP-problem.

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1. INTRODUCTION

There are good reasons to believe that all fundamental forces allow for a unified description. The running of the gauge couplings of strong and electroweak interactions and the symmetries of the particle content of the standard model (SM) are the most compelling ones and suggest a unified picture of these interactions through grand unified theories (GUTs) [1, 2]. The fundamental feature of these theories is that, at some high scale M_{GUT} , all gauge interactions of the SM are described by a single and bigger gauge group, such as SU(5), SO(10) or E_6 with a single (unified) gauge coupling. However, precision measurements at the weak scale indicate that within the SM the gauge couplings do not meet accurately. The situation is improved if supersymmetry (susy) is included. In the minimal supersymmetric extension of the SM (the MSSM), all gauge couplings meet (with acceptable accuracy) at $M_{GUT} \sim 3 \times 10^{16}$ GeV. Beside gauge coupling unification, GUTs offer natural explanations for neutrino masses, Yukawa unification and the structure of the matter generations.

A particularly interesting GUT is SO(10) [3, 4]. In this theory, one generation of quarks and leptons is elegantly accommodated within a single spinor representation of SO(10), according to the decomposition

where quantum numbers with respect to $G_{SM} = \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ are displayed. Remarkably, $\mathrm{SO}(10)$ GUTs predict the existence of right–handed neutrinos $\overline{\mathbf{v}}$ and include an additional $\mathrm{U}(1)$ (named $\mathrm{U}(1)_{B-L}$) that forbids dangerous dimension 4 proton decay operators.

Beside their attractive properties, GUTs introduce some problems of their own. The most puzzling feature is that, while matter generations are described by complete GUT representations (eq. (1)), SM Higgs and gauge bosons appear only as *split* (incomplete) GUT multiplets. Hence, additional fields are needed in order to obtain full GUT representations. However, these fields are problematic as they generically mediate fast proton decay. In the case of the SM Higgs, this is known as the *doublet-triplet splitting problem* and is present in all interesting GUTs. Apart from that, the breaking of GUT groups down to G_{SM} is rather involved and requires additional Higgs fields in large representations (e.g. $\overline{126}$ of SO(10)).

Some of these issues can be solved in higher-dimensional field theories compactified on orbifolds. Starting with a GUT group in 5 or 6D, the GUT symmetry breaking to G_{SM} is not induced by a Higgs mechanism, but by choosing appropriate boundary conditions for the gauge bosons in the extra dimension(s). Furthermore, by placing the three SM generations and the Higgs fields on different brane-like objects in the extra dimension(s), i.e. on fixed points of the orbifold, one can find a geometrical explanation for the difference between complete GUT matter representations and split Higgs multiplets [5, 6, 7].

These features studied in field theory orbifolds are also proper to compactifications of string theory. In particular, the chain of GUTs

$$G_{SM} \subset SU(5) \subset SO(10) \subset E_6 \subset E_7 \subset E_8$$
 (2)

suggests that $E_8 \times E_8$ heterotic orbifolds [8, 9] are natural candidates to provide an ultraviolet complete theory that solves the inherent problems of GUTs.

Heterotic orbifolds can be seen geometrically as singular limits of 6D smooth Calabi-Yau spaces (see fig. 1) and lead to models that resemble the MSSM [10, 11, 12, 13] (for details on their construction, see e.g. [14, 15, 16, 17]). In the transition to the orbifold, the complicated Calabi-Yau reduces to a simple space that is flat everywhere except for some isolated singularities, the so-called fixed points. This results in symmetry enhancements of two kinds: first, discrete symmetries with geometrical origin arise, and second, the gauge symmetry is enhanced. Both features are desirable from the SM perspective: discrete symmetries can provide answers to prevailing puzzles of low energy phenomenology, such as the strong CP-problem as discussed in section 3.1, and the 4D SM gauge symmetry can be enhanced locally at the various singularities to various GUTs. As described in the next section, the last feature gives rise to the concept of *local GUTs* [11, 12, 18], which has proven to be useful in heterotic orbifolds and recently also in F-theory [19].

2. LOCAL GRAND UNIFICATION

The concept of local grand unification in heterotic orbifolds is based on the observation that the $E_8 \times E_8$ gauge symmetry of the 10D bulk is broken only at the orbifold singu-

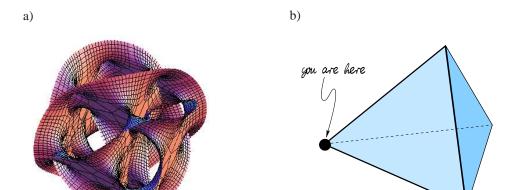


FIGURE 1. a) Calabi-Yau spaces versus b) orbifolds. Orbifolds are singular limits of Calabi-Yau spaces with a much simpler structure. Whereas studying the phenomenological properties of Calabi-Yau compactifications is very hard, many appealing features arise naturally in orbifolds suggesting that we might actually live close to an orbifold fixed point rather than a generic point in Calabi-Yau moduli space [20].

larities to various subgroups, giving rise to so-called local GUTs. In this section, we discuss this concept in more detail and show that local GUTs seem to be a necessary link between the heterotic string and the MSSM.

2.1. Local GUTs in Heterotic Orbifolds

Compactification of the heterotic string on orbifolds remains a simple and elegant method to achieve 4D theories that include chiral fermions and phenomenologically viable gauge groups. Orbifolds emerge from dividing a manifold by one of its discrete symmetries. We focus here on 6D toroidal abelian orbifolds, i.e. on orbifolds resulting from moding a discrete \mathbb{Z}_N symmetry out of a 6D torus. The whole curvature of the emerging space is concentrated at the points that are left fixed under the action of the generator ϑ of the \mathbb{Z}_N symmetry (from the 10D point of view, these points are brane-like objects of 4 or 6 dimensions). Internal consistency of the theory demands ϑ to be associated to an operation in the $\mathbb{E}_8 \times \mathbb{E}_8$ gauge degrees of freedom which, in the bosonic formulation used here, is encoded in a 16D shift vector V. In presence of Wilson lines [21], the gauge embedding varies at different localizations in the compact space, i.e. it is different from brane to brane. Thus, it is necessary to introduce the local shift V_{local} that parametrizes the orbifold action on the branes.

In heterotic orbifolds, massless states originate from closed strings of two types. The untwisted or bulk states are free to move in the whole space and stem directly from the strings associated to the 10D supergravity and $E_8 \times E_8$ vector multiplets. The 4D gauge bosons belong to this category. In addition, there are other states of pure stringy origin, the so-called twisted states. The corresponding strings close only thanks to the \mathbb{Z}_N symmetry, what constrains them to be attached to the associated fixed points. These states are matter fields in the 4D effective theory.

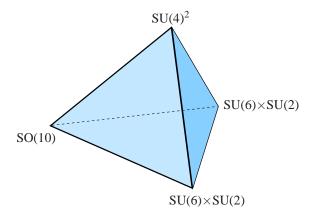


FIGURE 2. Gauge group topography of a \mathbb{Z}_2 orbifold. In heterotic orbifold compactifications, the bulk gauge group is $E_8 \times E_8$ and is broken at the singularities (corners) to local GUTs. The 4D gauge group, i.e. the common intersection of the local ones, is in this example $SU(3)_c \times SU(2)_L \times U(1)_Y$.

In a second step, bulk and twisted states are affected by the action of the orbifold. They acquire phases that depend on ϑ and its (local) gauge embedding V_{local} . From the perspective of a fixed point, only those states that are invariant under the local orbifold action remain massless. Omitting the details for the sake of brevity, this implies that the only gauge bosons surviving the local orbifold projection are those that satisfy

$$e^{2\pi i p \cdot V_{\text{local}}} = 1 \Leftrightarrow p \cdot V_{\text{local}} = 0 \mod 1,$$
 (3)

where p denotes the roots of the unbroken local gauge group. It follows then that, at each fixed point of the orbifold there is a local gauge group $G_{local} \subset E_8 \times E_8$ that is larger than the one in 4D (see fig. 2). The effective low energy gauge group is the intersection of some or all G_{local} , depending on the size of the internal space. For example, if all internal dimensions are of the order of the inverse string scale, the 4D gauge group is (in the absence of freely acting Wilson lines) the common subgroup to all local groups,

$$G_{4D} = \bigcap_{f} G_{\text{local},f} , \qquad (4)$$

where f runs through all fixed points. On the other hand, if two dimensions are larger (of the order of the inverse GUT scale), the effective gauge group of the resulting 6D effective theory is given by the intersection of the local groups associated to the fixed points within the four smaller dimensions. In some cases, this scenario can be interpreted as a 6D GUT which may break down to the SM in 4D. Another relevant observation is that twisted states located at the fixed points transform as complete representations of $G_{local,f}$, e.g. if the up-quark sits at some fixed point with local SO(10) gauge group, it has to be accompanied by the other quarks and leptons to form a full 16—plet.

This favors SO(10) local GUTs in models with $G_{4D} = G_{SM}$. If a **16**-plet is found on a special GUT brane with local SO(10) where the orbifold projection acts trivially, it appears in 4D as a complete generation of quarks and leptons. The Higgs fields, on the other hand, must not live on the same brane. They could be bulk states, transforming as $4D SU(2)_L$ doublets but not as full **10**-plets of the local GUT.

TABLE 1. Local GUT structure of the MSSM candidates. The SU(5) local GUT does not produce a complete family, so additional "non-GUT" states are required.

Local GUT	"Family"	2 Wilson lines	3 Wilson lines
E_6	27	14	53
SO(10)	16	87	7
SU(6)	$15+\overline{6}$	2	4
SU(5)	10	51	10
non GUTs		39	0
total		193	74

2.2. Fertile Patch in the Mini-Landscape

As soon as some 4D models with realistic properties were found from \mathbb{Z}_6 –II heterotic orbifolds [11, 12], it was realized that they were endowed with local GUTs. This seems to hint towards a selection principle: models with realistic properties are intimately attached to local GUTs. If this principle is indeed true, finding the MSSM vacuum from string theory reduces to spotting first the regions of the huge string landscape [22, 23, 24] endowed with local GUTs.

How accurate the previous selection principle is can be judged only by taking it seriously as a guiding strategy in the search for realistic vacua. This is the approach followed in the Mini-Landscape (ML) [25, 26, 27], based on \mathbb{Z}_6 -II orbifolds (for details on \mathbb{Z}_6 -II, see e.g. [11, 28]). The strategy is simple: one must focus on models endowed with local GUTs at special points where the orbifold projection is trivial and impose a series of phenomenological constraints. In the ML, we have demanded a) $G_{4D} = G_{SM}$ with local SO(10) or E_6 GUTs; b) gauge coupling unification ($\sin^2 \theta_w = 3/8$) with nonanomalous hypercharge from SU(5); c) three SM matter generations; and d) vectorlike exotics x_i whose masses arise from couplings to SM singlets that acquire VEVs in susy vacua. The ML search results in $\mathcal{O}(100)$ promising MSSM models, supporting the strength of the local GUT strategy.

Naturally, one might argue that there are perhaps other random regions of the \mathbb{Z}_6 –II string landscape far from the local GUT patch that yield comparable vacua. In order to discard this, we have performed independently a random search [27]. The results are listed in table 1. First, out of a total of $\sim 10^7$ models we have identified more than 200 hundred MSSM candidates. Second, we notice that most of the MSSM candidates have two Wilson lines, resulting in a D_4 family symmetry where two generations form a doublet and the third a D_4 singlet. Third, most of the promising models contain E_6 or SO(10) or SU(5) local GUTs.

These results indicate that a search strategy based on local GUTs leads to the vast majority of the phenomenologically viable models. That is, as opposed to a random search, local GUTs are ideal benchmarks to pinpoint the regions of the string landscape where the vacuum that describes our universe may be found.

Supersymmetry Breaking, R-Parity and Seesaw Neutrinos. The ML models have many further appealing features. First, it is found that susy is preserved at high energies as long as there is a subset of SM singlets s_i that acquire VEVs close to the string scale. This breaks many of the additional gauge factors in the 4D gauge group, but can leave certain nonabelian hidden gauge groups unbroken. In the MSSM candidates, it turns out that these hidden sectors are appropriate for gaugino condensation and lead to susy breaking with $m_{3/2} \sim \text{TeV}$ [29].

In order to prevent the appearance of dangerous, dimension-four lepton (L) and baryon (B) number violating operators that would allow for too rapid proton decay (e.g. $\bar{u} \bar{d} \bar{d}$ and $q \ell \bar{d}$), the MSSM includes R-parity (or matter parity) [30]. In some ML models, matter parity could be identified from the spontaneous breaking of U(1) $_{B-L} \not\subset SO(10)$ by the VEVs of some s_i with even charges [26],

$$U(1)_{B-L} \xrightarrow{s_i} \mathbb{Z}_2^{\text{matter}} = (-1)^{3(B-L)}. \tag{5}$$

Notice that the existence of such singlets is associated with the fact that $U(1)_{B-L}$ turns out to be a mixture of the standard B-L from SO(10) and other U(1) factors. This symmetry allows the distinction between Higgs h_d (even parity) and the leptons ℓ (odd), and between standard model singlets s_i (even) and right–handed neutrinos \overline{v}_j (odd). The promising ML models exhibit $\mathcal{O}(100)$ \overline{v}_j 's and the couplings

$$W \supset Y_{\nu}^{ij} h_{u} \ell_{i} \overline{\nu}_{j} + \frac{1}{2} M_{jk} \overline{\nu}_{j} \overline{\nu}_{k} , \qquad (6)$$

where Y_{v}^{ij} and M_{jk} are polynomials of the VEVs of s_{i} . This implies that the seesaw mechanism is quite generic in orbifold models with realistic properties and the seesaw scale is slightly reduced due to the large number of right-handed neutrinos [31].

Another generic feature of the ML MSSM candidates is the exceptional position of the top quark: only for the top quark there is a trilinear Yukawa coupling. In addition, this coupling originates from the $E_8 \times E_8$ gauge coupling in 10D and hence the coupling strength y_t is given by the gauge coupling g at the unification scale. This equality receives corrections due to localization effects of bulk fields in the presence of localized Fayet-Iliopoulos terms that favor a slight suppression of y_t against g [32].

From these properties, it seems natural to conclude that \mathbb{Z}_6 -II heterotic orbifolds furnished with local GUTs represent a fertile framework for producing models connecting string theory with the MSSM.

3. ACCIDENTAL SYMMETRIES

Similar to the appearance of accidental *B* and *L* number conservation in the standard model, global accidental symmetries can arise in string theory models as follows. 10D string theory offers a wide range of exact symmetries: apart from gauge symmetries, there are (target-space) modular and continuous space-time symmetries. Due to the compactification to 4D, these symmetries are generically broken to a multitude of subgroups of various types. For example, for heterotic orbifolds the breaking of the 6D part of the 10D space-time symmetry results in some discrete (R-)symmetries [33, 34].

The unbroken symmetries restrict the form of the superpotential of the effective 4D theory and can be of great phenomenological relevance, e.g. as family symmetries [35] or matter parity [36]. They could also provide an explanation of the stability of the proton [37, 38].

Additionally, focusing our attention on superpotentials of a limited degree in the fields, global accidental symmetries can arise [39] that are explicitly broken to the exact discrete symmetries at higher orders in the superpotential. Nevertheless, some of these accidental symmetries, can remain unbroken up to very high orders and might be important to explain some of the issues of low energy physics such as the origin of large hierarchies in connection to the susy μ -term problem [40], moduli stabilization or the strong CP-problem, as will be discussed in the following.

3.1. Accions

The most elegant and appealing solution to the strong CP-problem is based on the conjecture of an axion field [41, 42]. It requires the existence of an anomalous global Peccei-Quinn symmetry [43] $U(1)_{PQ}$ and its spontaneous breakdown at a scale F_a (where F_a denotes the axion decay constant). Constraints (mostly) from astrophysics and cosmology require F_a to be in the axion window

$$10^9 \,\text{GeV} \le F_a \le 10^{12} \,\text{GeV} \tag{7}$$

for the so-called "invisible" axion [44, 45]. The axion field adjusts its VEV to cancel the θ -parameter of quantum chromodynamics (QCD) to avoid CP-violation due to strong interactions.

In the heterotic string, there are various sources for axions. The spontaneous break-down of the anomalous U(1) together with the Green-Schwarz mechanism produce the so-called model-independent axion, whose decay constant is fixed by the Planck scale (see e.g. [46]), too high to solve the strong CP-problem. Model-dependent axions arise from the internal components of the B-field. Unfortunately, their decay constants are in general not much lower than in the previous case [46]. Admissible axions, on the other hand, can arise from the breaking of (multiple) anomalous accidental global U(1) symmetries realized as low energy remnants of (stringy) discrete symmetries [47]. These axion-like particles are referred to as *accions* [48].

The accion decay constant depends on the VEVs of the fields responsible for the breakdown of such global U(1)s. In general, if there are N singlets acquiring VEVs and M global accidental U(1)s broken spontaneously at hierarchically different scales, then F_a is of the scale of the Mth largest VEV.

It follows that in specific vacua in which some accidental U(1)s are broken at an intermediate scale (e.g. $10^{12}~\text{GeV}$) a QCD accion decay constant satisfying current constraints can be achieved. This situation is indeed realized in specific vacuum configurations of MSSM orbifold candidates [48]. It has been additionally verified that the contributions to the accion mass due to the explicit breaking of the accidental U(1)s can lie in the admissible experimental window.

4. CONCLUSIONS

We have reviewed the concept of local GUTs and shown how models with local GUTs can alleviate some of the disadvantages of traditional GUTs, such as the doublet–triplet splitting problem. Moreover, this concept represents arguably one of the most successful benchmarks to pinpoint the region of the immense string landscape where the vacuum that describes our universe may be found. In particular, in the context of \mathbb{Z}_6 –II orbifolds, it leads to a large set of MSSM candidates with the exact spectrum of the MSSM, coupling unification, seesaw neutrino masses and low energy susy breaking.

We have also studied some of the consequences of the various (global approximate) symmetries that appear naturally in string compactifications as remnants of stringy discrete symmetries. These symmetries could provide an explanation of the origin of the huge hierarchy between the Planck and electroweak scales. Such hierarchies are also important for moduli stabilization. In addition, accidental symmetries lead to natural candidates for QCD axions.

As a general result, explicit string constructions towards the MSSM (and SM) seem to require local grand unification and a very special location of fields in extra dimensions. We do not seem to live at a generic point in Calabi-Yau moduli space but rather at a point of concentrated curvature that can be described very well by an orbifold fixed point. Location of the fields close to that fixed point leads to additional (discrete or accidental) symmetries which are relevant for specific properties of the MSSM.

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